Measuring the Absolute Volume Scattering Function with LISST-100

SEQUOIA SCIENTIFIC, INC. ✦ APPLICATION NOTE L012

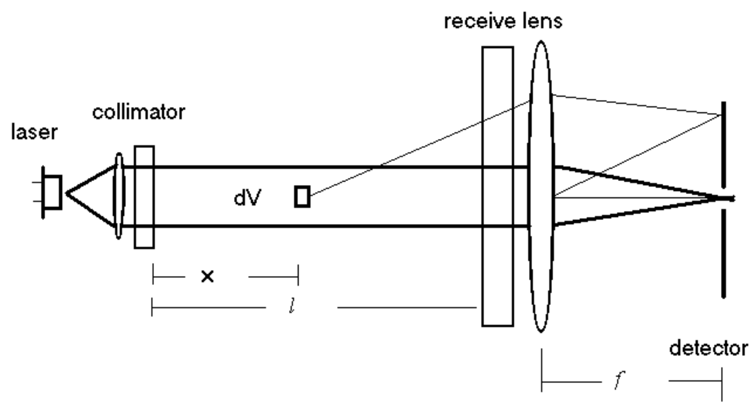

Figure 1: A collimated laser beam (left) illuminates particles between the 2 windows. A multi-ring detector placed at the focal plane of the receiving lens senses scattered light. The figure also illustrates the Fourier transform property of a lens, namely that any ray originating at an angle θ from the optical axis reaches the detector at a radius equal to $f\theta$.

This note describes the formulation that relates the measured multi-angle scattered light to the VSF of water.

Figure 1 shows the basic optical geometry.

Consider a small length dx of the beam, located at a distance x from the transmit window. An elementary volume in the laser beam is $dV=dA\ dx$ where dA is the area of the elementary volume. Let $\beta(\theta)$ be the VSF. Let the optical path of the beam between the windows be l. For optical power P_o entering from the transmit window, and with a beam attenuation coefficient c, by definition of the VSF, scattered power in any direction θ will be:

$$dP = e^{-cx}\, P_o/A\ \beta(\theta)\ dA\ dx\ d\Omega \qquad (1)$$

where $d\Omega$ is the elementary receiver solid angle. The exponential factor at the front of the right hand side of equation (1) accounts for beam power attenuation to the distance x. Further attenuation of scattered light occurs by a factor $c^{-c(l-x)}$ before reaching the receive window[1]. The solid angle $d\Omega$ is dS/f^2 where dS is area of an elemental ring on a detector that senses light scattered over angle θ to $\theta+d\theta$, and f is the receiving lens focal length. Substituting,

$$dP = e^{-cl}\, P_o/A\ \beta(\theta)\ dA\ dx\ dS/f^2 \qquad (2)$$

But, since any ring detects light scattered at the same scattering angle,

$$dS = 2\pi\phi f^2 \theta\, d\theta$$

where ϕ is the fraction of a circle covered by a detector ring. Substituting and integrating, we have the power on ring number i as

$$P_i = \iiint e^{-cl}\, P_o/A\ \beta(\theta)\ dA\ dx\ 2\pi\phi\theta\, d\theta. \qquad (3)$$

The triple integral is over the cross-section, the length, and the angles for each ring. Now, if VSF is assumed

[1] The exponent should formally be written as $e^{c(l-x)/\cos(\theta)}$. The form that is used applies to small θ.

to be independent of the location of the elementary volume in the water, and if we further assume that VSF is a slowly varying function of angle so that it remains constant over the small angle sub-range covered by each ring, then this is integrated over to:

$$P_i = e^{-c\,l}\, l\, P_o\, \pi\phi\, \beta(\theta)\ [\theta^2_{i+1} - \theta^2_i]. \qquad (4)$$

Now, the first exponential factor is recognized as optical transmission τ. Since, there are 32 detectors spanning a radius range 200:1, and detector radii increase logarithmically, if ρ represents the factor $200^{1/32}$,

$$\theta_i = \rho^{i-1}\ \theta_{min} \qquad (5)$$

so that, we have after substitution and some algebra

$$\beta_i(\theta) = [P_i / P_o] / [\ (1-\rho^{-2})\,\rho^{2i}] / [\pi\, l\, \phi\, \tau\, \theta^2_{min}\,] \qquad (6)$$

This is the essential relationship between the power sensed by each silicon ring detector and the VSF averaged over it.

All quantities in the above formulation are measured with the LISST-100, though the precise magnitude of the laser power P_o and scattered power P_i requires the responsivity of the detectors. Let the photo-detector response be such that the associated post-amplification voltage is:

$V_i = G_i\, P_i$, and

$V_o = G_o\, P_o$

where G is the gain, (Volts/Watt-optical-power).
It follows that:

$$\beta_i(\theta) = G_o/G_i\ \ V_i / V_o / [\ (1-\rho^{-2})\,\rho^{2i}] / [\pi\, l\, \phi\, \tau\, \theta^2_{min}\,] \qquad (7)$$

The purpose of writing the result in the above form is to show that all quantities are measured, except the responsivities G. Values of G can be obtained from standards that are traceable to the US National Institute of Standards and Technology, NIST. With a common A/D, the ratio V_i / V_o can be replaced with the ratio of the corresponding digital counts, i.e. N_i/N_o[2].
Thus the task of measuring the VSF reduces to recording the voltages V_i and V_o. The LISST-100 instrument records V_i from the 32 ring detectors following an *I-V* amplifier, using an overall gain of 0.5V/µW-optical[3]. The corresponding sensitivity for the transmission sensor is ~1mV/µW-optical. As a final form, then, for the assumed values of parameters in this example:

$$\beta_i(\theta) = 2.0\ 10^{-3}\ N_i / N_o / [\ (1-\rho^{-2})\,\rho^{2i}] / [\pi\, l\, \phi\, \tau\, \theta^2_{min}\,] \qquad (8)$$

The VSF is obtained at the center of the angles covered by each ring detector. As a final comment, we note that the slope of the VSF at the smallest angle is constrained by the largest particles that may be present in water. For more details, please contact us.

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[2] The laser power output is measured using a reference photo-diode. This diode senses power in a portion of the beam split from the original laser output beam. Thus the laser output is calibrated as number of counts of the reference-diode per mW of in-water laser power.

[3] This parameter may vary. Please consult factory. In some units the value is twice higher, in others up to 100X.